

Mathematical Solution for Average Metering Error of Domestic Water Meters – a Relative Error Method

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Abstract

As everyone knows that domestic water meters have metering errors which cause under registration in average. However almost no one can be sure what the figure is, because the metering error is affected by two complicated behaviors: water consumption of household, and the inaccuracy of meter itself. And it is hard to gain enough data for studying both of them. Actually, the two behaviors are already contained within historical meter readings. This research analyzed the meter readings and developed a practical approach to identify average metering error for water utilities. Using the $R_M - \mu_z$ chart with the equations comprising statistical parameters, Taipei Water Department (TWD) has successfully estimated average metering error in its 99.6% domestic meters, including Sub Meters (-9.85% in average, under registration), Master Meters (-6.11% in average, under registration) and Direct Meters (-10% in average, under registration). Due to TWD's specific billing mechanism among Master Meters & Sub Meters, the equivalent metering error of Master-Sub meter groupings is only -4.26% in average. And its metering performance is better than any other non-grouping meters. Additionally, this method may provide an alternative way to estimate the Customer Meter Inaccuracies term in Water Balance Sheet.

Keywords

Metering Error; Meter Error Curve; Meter Readings; Relative Error; Random Variable

INTRODUCTION

Water revenue is the main income for the utility. TWD has over 1.59 million water meters in service and they are the only basis for water registration. It is understood that any water meter has metering error in nature, and it changes with type, size, age and condition of use. The only thing we're sure is that they are always under registering than they should be. But what is the average metering error which results in under registration? It is the concern of every single utility around the world.

Generation of Metering Error

Metering error of a water meter is determined jointly by user's consumption behavior (Intake Spectrum) and metrological characteristic of water meter (Meter Error Curve). Therefore, Meter Error Curve can not be singly regarded as metering error itself. The main factors affecting the Intake Spectrum is the habit of water consuming from customer which is the most unknown part of all. Fig. 1 shows that there will be a different metering error with different intake behavior, although they are the same water meter with the same character. Therefore if it is desired to know metering error, the water consuming behavior of the customers shall be taken into account.

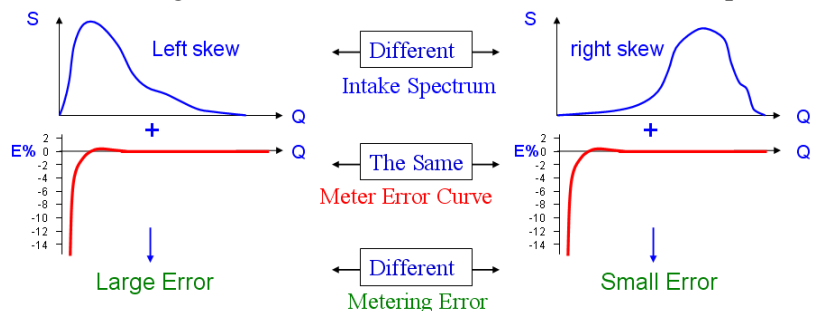


Fig. 1. Two major factors of metering error

Overall Metering Error

Water utilities and academies have not come to an agreed useful method of evaluating average meter error for overall meters. Some of them adopt Testing Method to estimate metering error. However, the cost for testing is too high that only limited samples can be tested. It is impossible to lunch a full-scale testing project to obtain entire metering error and it turned out to be case study which is generally a poor interpretation to overall behavior. The fatal defect of testing method is that it can not simulate the water consuming behavior of the customers in the lab. It means testing approach can only obtain Meter Error Curve, not metering error at all. Therefore, Testing Method can be applied only on small amount of meters, but improper for estimating the overall metering error of a water company whose meters in the number of, sometimes, 100 thousand or more. This research uses TWD billing system’s Historical Meter Reading Records which actually contain both water consumption behaviors and meter characteristics, so the outcome is better than Testing Method and the estimated metering error would be unbiased.

METHODS

Relative Error of Master-Sub Meter Groupings

Data used in this research came from TWD Meter Reading Records of “Master-Sub Meter Grouping”. TWD has 120 thousand such groupings, each grouping produces 6 reading records for one year, and there are about 720 thousand records in a year. The readings of Master Meter are given as R_M and the sum of Sub Meter readings are given as R_m shown in Fig.2.

The Metering Error of Master Meter (X) and Sub-Meter(Y) are as the following:

$$X = \frac{R_M - R}{R}, Y = \frac{R_m - R}{R} \dots\dots\dots (1)$$

Since the real water consumption R is unknown, so both X & Y can not be determined directly. In this research, we introduced relative error Z, which can be determined, because R_M and R_m are both meter readings which are not unknown. Z is defined as below:

$$Z = \frac{R_m - R_M}{R_M} \dots\dots\dots (2)$$

Place (1) into (2) and obtain the relations among X, Y & Z

$$Z = \frac{Y - X}{X + 1} \dots\dots\dots (3)$$

Since X, Y & Z are all random variables, the values change randomly, so the individual data of X, Y or Z can not reflect the overall behaviour, and only their statistical data, μ_x and μ_y , can describe the behaviour of average metering error (μ_x is the average metering error of Master Meters and μ_y is the average metering error of Sub Meters). Once you get those statistical values, you know the overall behaviour. So, acquiring those statistical values becomes the main job of this research. For obtaining the statistical values, a common way is to take Expected Value on both sides of

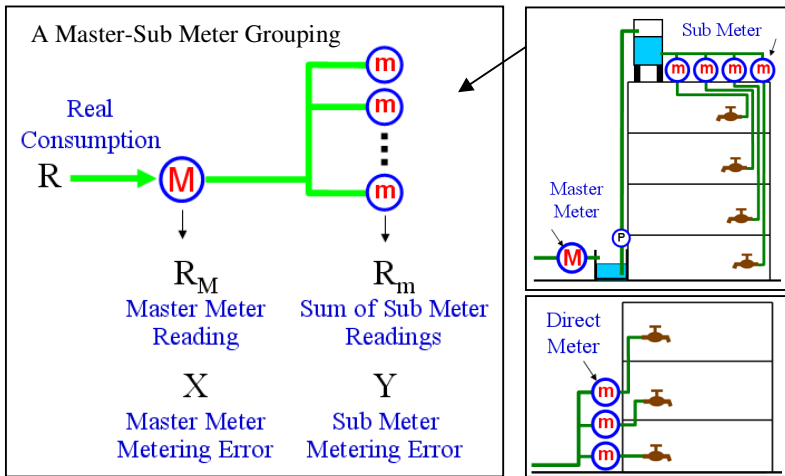


Fig.2. Master-Sub Meter Grouping and Direct Meter

equation of random variable. By doing this on eq.(3),

$$E[Z] = E\left[\frac{Y - X}{X + 1}\right] \dots\dots\dots (4)$$

However, $\frac{Y - X}{X + 1}$ is a non-linear function, it is difficult to derive the closed-form Expected Value from $E\left[\frac{Y - X}{X + 1}\right]$. To solve the non-linear problem, here using the Taylor Series by expanding $\frac{Y - X}{X + 1}$ at $X = \mu_x, Y = \mu_y$, and each term will turn into linear. Neglecting high order terms and taking Expected Value for a proximate derivative (Walpole et al., 2002), then we can reach the relationship between μ_x, μ_y, μ_z , that is,

$$\mu_z = \frac{\mu_y - \mu_x}{\mu_x + 1} + \frac{\mu_y + 1}{(\mu_x + 1)^3} \sigma_x^2 \dots\dots\dots (5)$$

Similarly, after neglecting the high order terms, if not taking the Expected Value $E[\dots]$, but taking the Variance Value $\text{Var}[\dots]$, then we get:

$$\sigma_z^2 = \frac{(\mu_y + 1)^2}{(\mu_x + 1)^4} \sigma_x^2 + \frac{1}{(\mu_x + 1)^2} \sigma_y^2 \dots\dots\dots (6)$$

Where $\mu_x = E[X], \mu_y = E[Y], \mu_z = E[Z], \sigma_x^2 = E[(X - \mu_x)^2], \sigma_y^2 = E[(Y - \mu_y)^2], \sigma_z^2 = E[(Z - \mu_z)^2]$

Although μ_z, σ_z are known (Each Z value can be determined by eq.(2). Its mean value μ_z and standard deviation σ_z are easy to calculate). However, as μ_x, σ_x and μ_y, σ_y are unknown, while there are 2 equations (5) & (6), but there are 4 unknowns, it is necessary to introduce other constraint conditions to cut down the unknowns to be able for solutions.

Employing Constraint Conditions

All TWD water meters whose diameter less than 50mm are Class B “Multi-Jet” wet type mechanical meters and their metrologies are featured with large metering error in low flowrate. However, the Error Curve of this type of meter has an apparent linear section at high flowrate and the metering accuracy in this area is quite high. According to the specification of Class B, the metering error is within $\pm 2\%$ along this linear section,

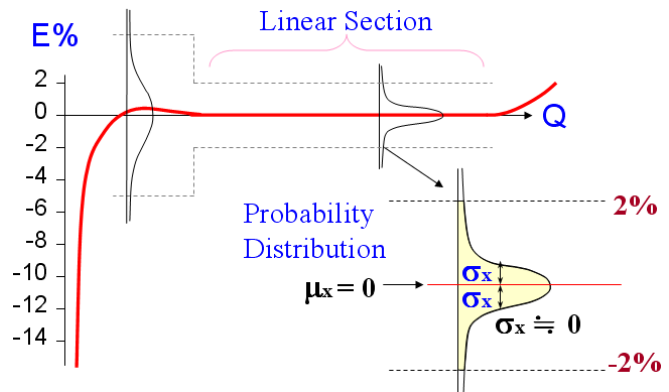


Fig.3. Constraint Conditions from Master Meter's Linear Section

as shown in Fig. 3. That means the average metering error $\mu_x = 0$ and standard deviation $\sigma_x \approx 0$ within the linear section. This is the constrain conditions. When Master Meters are under high flow, the statistical value of X will converge to $\mu_x = 0, \sigma_x \approx 0$. Since the metering error of Sub Meter (Y) and the metering error of Master Meter (X) are independent from each other, Y is not affected by X, so that the Sub Meter statistical values μ_y, σ_y will not be changed while the adoption of constraint conditions of Master Meters. Only that the X is selected at the high flow may be regarded as “sampling” and the obtained μ_y, σ_y must be put a “cap” above them to represent the estimated values $\hat{\mu}_y, \hat{\sigma}_y$.

Place the constraint conditions $\mu_x = 0, \sigma_x \approx 0$ and the known μ_z, σ_z into equation (5) & (6), they can be simplified into the following equations:

$$\hat{\mu}_y \approx \mu_z \dots\dots\dots (7)$$

$$\hat{\sigma}_y \approx \sigma_z \dots\dots\dots (8)$$

Now the estimated value of the average metering error of Sub Meters can be determined. Yet, how to select Master Meters of high flow from the meter reading data? We shall bring to you a simple and powerful tool: $R_M - \mu_z$ Chart.

Estimating the Metering Error by $R_M - \mu_z$ Chart

Making of $R_M - \mu_z$ Chart. By setting several intervals of R_M from Master-Sub groupings, averaging all the Z values within each interval to obtain each μ_z , plotting these (R_M, μ_z) pairs, then the $R_M - \mu_z$ chart is done. As the example shown in Fig. 4, with setting 100 m^3 as an interval width and at the interval between 500 and 600, the average relative error μ_z is calculated as $\mu_z = -0.07$, then connect all the center points of each interval to make the chart simple and clear.

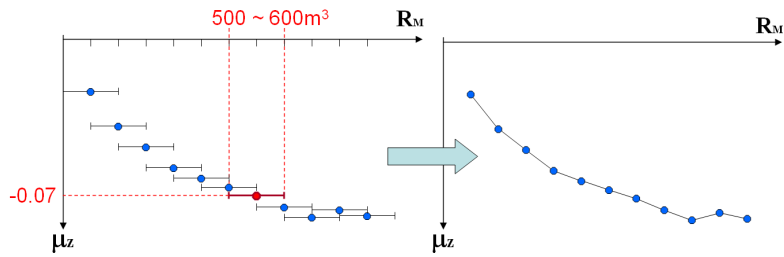


Fig.4. Plotting $R_M - \mu_z$ curve

Metering Error of Sub Meters. Taking 40 mm Master Meters for example, if we consider only the water meters aged within 5 years (excluding the aged meters) and draw up the $R_M - \mu_z$ in the way prescribed above, then, as shown in Fig. 5, there is an obvious flat zone of R_M in the range between 2100 and 3100 m^3 and μ_z is around -0.1. Within the flat zone, the intake flows of Master Meters fall within the linear section of Meter Error Curve, and subject to the constraint conditions (i.e. $\mu_x = 0, \sigma_x \approx 0$).

If aged meters older than 5 years are excluded, and plot all the Master Meters of different sizes into $R_M - \mu_z$ Charts, as Fig. 6, their flat zones are apparently close to $\mu_z = -0.1$ and the curve profiles are pretty similar to each other. This is because the Master Meters of TWD from $\phi 20$ to $\phi 40\text{mm}$ are all of Class B wet type Multi-jet. However, Master Meters of $\phi 50$

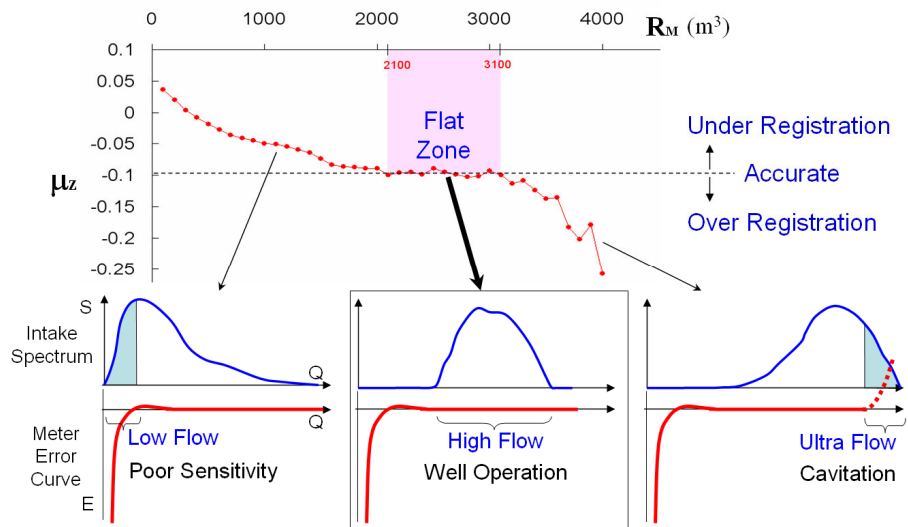


Fig.5. Flat Zone in $R_M - \mu_z$ curve subject to the constraint conditions

comprise many different types, including Multi-jet, horizontal or vertical Woltmann, wet or dry type, and mechanical or electronic type, etc. The Meter Error Curves of them are slightly different from one another. That's why the profile of $R_M - \mu_z$ Chart of $\phi 50$ is a little bit different from those of $\phi 20 \sim \phi 40$, but there is still a flat zone by $\mu_z = -0.1$.

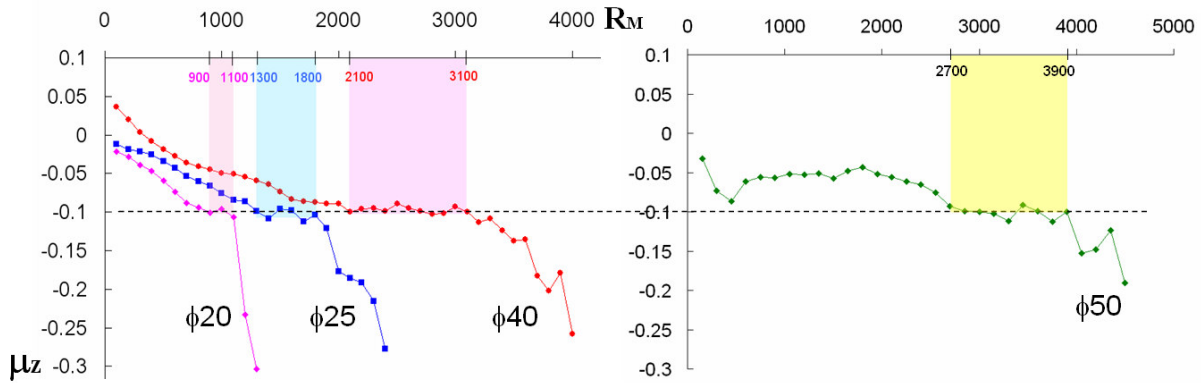


Fig.6. $R_M - \mu_z$ Charts of $\phi 20 \sim \phi 50$ Master Meters

Using $R_M - \mu_z$ Chart of Fig. 6, it is easy to pick up Master Meters of high flow within flat zone for calculating the values of μ_z, σ_z . Fit them with equation (7) and (8), we can find that $\hat{\mu}_y \approx \mu_z = -0.09854$, $\hat{\sigma}_y \approx \sigma_z = 0.15723$, so the average metering error of Sub Meters is -0.09854 , or -9.85% . (Under registration)

Metering Error of Master Meters. From last section, the values of μ_y, σ_y have been estimated as $\hat{\mu}_y = -0.09854$, $\hat{\sigma}_y = 0.15723$, and by using all the data of Master-Sub meter groupings, μ_z, σ_z can be calculated as $\mu_z = -0.02453$, $\sigma_z = 0.12350$. With equations (5) and (6), we can get solutions for μ_x, σ_x (2 equations with 2 unknowns). But since the exponents of μ_x, σ_x in equation (5) and (6) can be as high as 4, it is difficult to solve μ_x, σ_x . In order to work out this problem, we can shift the terms of $Z = \frac{Y-X}{X+1}$ to arrive at $X = \frac{Y-Z}{Z+1}$, and take Taylor series expansion of $\frac{Y-Z}{Z+1}$ around $Y = \mu_y, Z = \mu_z$, omit the high order terms and take expected value at both sides of equal sign, we will have the equations for calculating μ_x, σ_x directly:

$$\mu_x = \frac{\mu_y - \mu_z}{\mu_z + 1} + \frac{\mu_y + 1}{(\mu_z + 1)^3} \sigma_z^2 \dots\dots\dots (9)$$

$$\sigma_x^2 = \frac{(\mu_y + 1)^2}{(\mu_z + 1)^4} \sigma_z^2 + \frac{1}{(\mu_z + 1)^2} \sigma_y^2 \dots\dots\dots (10)$$

Place $\mu_z = -0.02453$, $\sigma_z = 0.12350$ into equation (9) and (10), then obtaining the estimated values $\hat{\mu}_y = -0.09854$, $\hat{\sigma}_y = 0.15723$

of Master Meter average metering error: $\hat{\mu}_x = -0.06106$, $\hat{\sigma}_x = 0.1992$

So, overall average metering error of Master Meters is -0.06106 , or -6.11% . (Under registration)

Metering Error of Direct Meters. We have calculated the overall average metering error of Master and Sub Meters. What will be the metering error of Direct Meters? As mentioned above, the metering error is depending on the Intake Behaviour and the Water Meter Characteristics. Sub Meter has similar intake behaviour as Direct Meter does, i.e. as long as the user turn on faucet, the meter has water intake, and the intake spectrum are both at left-skew distribution, which stands for the small volume intake behaviour. Also, either Sub or Direct Meter use the same water meter (Multi-jet, mechanical and wet type), the water meter characteristics are the same to each other. Therefore, it is quite clear that the metering error of Direct Meter will be very close to that of Sub Meter, around -10% . (Under registration)

RESULTS & DISCUSSION

Overall Metering Error

Metering Error of Master-Sub Meter Groupings. TWD had already applied a unique billing mechanism for a very long time. When a Master Meter reading is higher than the sum of its Sub Meters, water billing will be based on the reading of Master Meter; on the other hand, when the sum of Sub Meters readings is higher than their Master Meter reading, the readings of the Sub Meters will be based on billing.

It seems that TWD is always the winner and some of our customers truly believe that TWD is overcharging in long terms by taking advantage of this mechanism. Although the water tariff is not expensive, it still gives the public with poor image. Is it true that TWD will never at loss, or customers are actually gaining from the mechanism? If we can find out correct answer scientifically, we can then put away the complaint. This research converts all the Master-Sub Meter Groupings into an Equivalent Meter, as shown in Fig. 7 and the Equivalent Metering Error will be the answer, defined as following:

$$\varepsilon = \frac{\left(\sum^{Z<0} R_M + \sum^{Z \geq 0} R_m \right) - \sum R}{\sum R}, \text{ here } \sum R = \frac{\sum R_m}{1 + \mu_y} \dots \dots \dots (11)$$

We can obtain from the 2009 Meter Reading Records of Master-Sub Meter Groupings:

$$\sum^{Z<0} R_M = 98,773,322 \cdot \sum^{Z \geq 0} R_m = 52,247,504 \cdot \sum R_m = 142,201,224$$

According to last section, $\mu_y = -0.09854$. Place this into Equation (11), then we have $\varepsilon = -0.04263$

The Equivalent Metering Error of Master-Sub Meter Groupings is -4.26% (Under registration). Although it is more accurate than -6.11% of the average metering error of Master Meters, but it tells us that TWD is still short charging by 4.26%. While TWD seems to get the apparent advantage under the billing mechanism, but those who receive the actual benefits are TWD's customers.

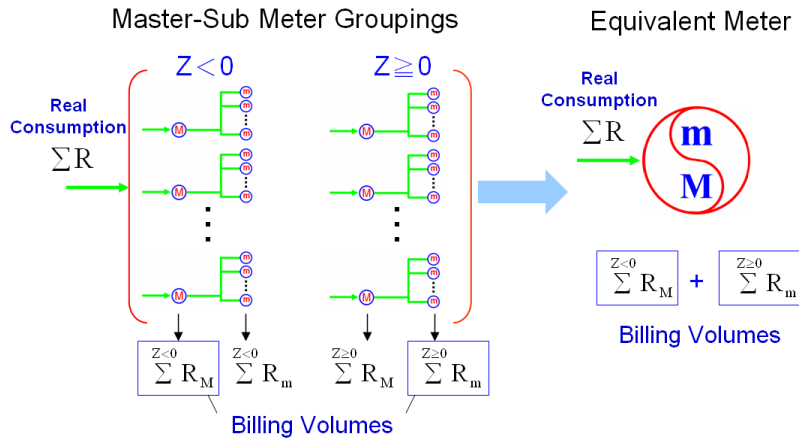


Fig.7. Master-Sub Groupings Converting to the Equivalent Meter

| | Metering Error |
|---------------|----------------|
| Master Meters | -6.11% |
| Sub Meters | -9.85% |
| Direct Meters | -10% |

| | |
|----------------------------|----------------------------------|
| Master-Sub Meter Groupings | Equivalent Metering Error=-4.26% |
| Overall Meters | Overall Metering Error=-5.90% |

Fig.8. Metering Errors of Different Groups (year 2009)

Estimation of Overall Metering Error. TWD's average metering error of water meters can be divided into two categories, one is estimable and the other is not estimable. The estimable are all small-diameter ones, in total of 1.58 million and accounts for 99.6% of all the water meters. The other is large diameter water meters whose average metering error are not estimable, because their metrological characteristics and water intake behaviours are far from small diameters of which this research based upon.

Fortunately, large meters only accounts for 0.4% of all, we still can estimate Overall Metering Error just by small meters. Using -10% of average metering error of Direct Meters and -4.26% of Master-Sub Meter groupings, weighted by its sum of meter readings, then we get the average Overall Metering Error of -5.90% (Under registration). Each Metering Error mentioned above is listed in Fig. 8.

$R_M - \mu_z$ Chart: An Inspection Tool for Meters

Meter Performance of different size. Combining $R_M - \mu_z$ Chart with the distribution of meter readings in Fig.9, it is quite clear that almost all meters' readings concentrate on the under registration side. There are only less than 1% of them are locating at the accurate or over registration side. Among them, $\phi 40$ Master Meter is even worse, its $R_M - \mu_z$ curve has a section with μ_z higher than 0, which means the registration volumes of Master Meter are less than its Sub Meters. By analyzing the area under the meter readings distribution of $\phi 40$, there is 45% area locating higher than $\mu_z = 0$. It tells that $\phi 40$ is the poorest performer. This is mainly because TWD do not use $\phi 32$ mm domestic meters, small water consumption registered by larger $\phi 40$ mm meter instead, causing so called "Metering Gap" between $\phi 25$ and $\phi 40$. If we look at the curve for $\phi 50$ at right side of Fig. 9, the behaviour is totally different- the $R_M - \mu_z$ curve is not bending up at small R_M , and the error is not magnified uncontrollable. This shows that $\phi 50$ is the better performer. That is because TWD's $\phi 50$ Master Meters contain not only Class B but also some electronic Class C meters which are more sensitive under small flow.

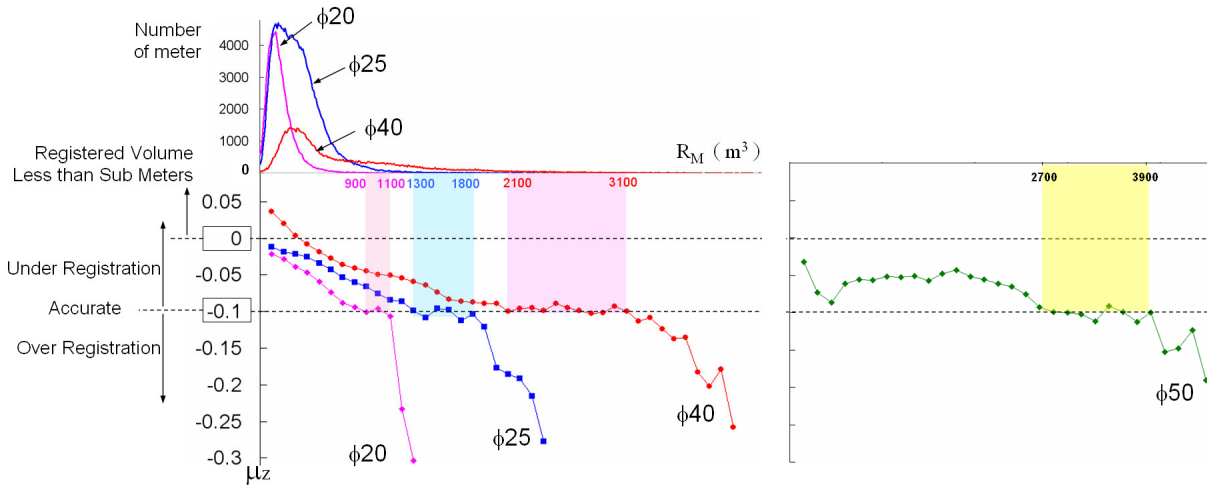


Fig.9. Inspection of Performance of Meters

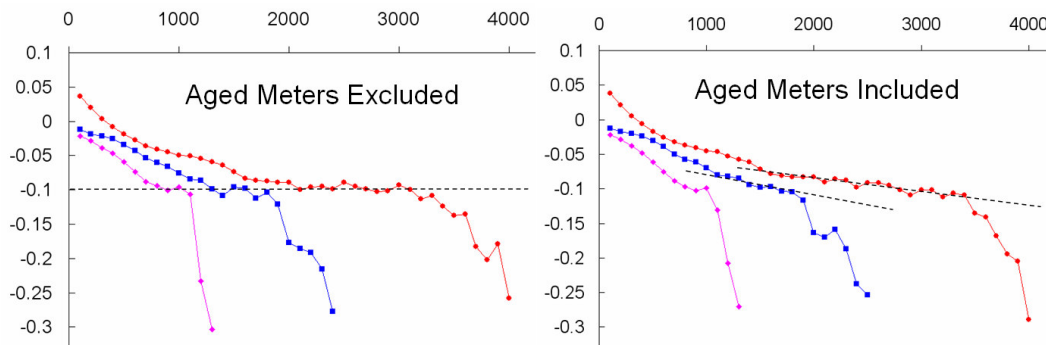


Fig.10. Influence of Aged Meters

Influence of Aging. After using for long time, water meter will suffer from increasing resistance to the moving parts due to limescale built-up and deformation of components. The Mechanical Meter has further gear stuck problem, which reduce the sensitive of metering and enhance the trouble of water registration. So when the meter is aged, the metering accuracy will be degraded as well. The error will increase while metering the small flow. If the aged Master Meters were not excluded, the $R_M - \mu_z$ curves will be like the right chart of Fig. 10, with inclined Flat zone and resulted in

difficulty to estimate $\hat{\mu}_y, \hat{\sigma}_y$ with well precision. It is discovered in this research that as long as the meter older than 5 years is excluded, the flat zone of $R_M - \mu_Z$ curves will be perfectly horizontal and $\hat{\mu}_y, \hat{\sigma}_y$ will be estimated without error.

CONCLUSIONS

Monitoring Overall Accuracy or Health of Meters

Most utilities around the world have adopted the IWA Water Balance Sheet for auditing their water consumption and losses. One component in the Sheet is named “Customer Meter Inaccuracies” which is most unknown term and constantly changing with time. It affects the outcome of evaluation for real losses, sometimes resulting in the wrong estimation of water leakage. This research provides a solution to estimate this unknown changing term with an inexpensive way by analyzing the meter reading records. Furthermore, through a long-term observing overall metering error will provide a great help for monitoring the health of meters. Fig. 11 shows that

TWD’s overall metering error in the past 6 years was slightly getting better. It reflects that TWD’s meter replacement work is adequate which maintains the overall health of meters in a good manner. TWD is currently studying the introduction of Class C meter. This research will be a useful tool for monitoring whether the adoption of Class C does reduce the overall metering error, and whether the increase of revenue meets the expenditure of meter replacement.

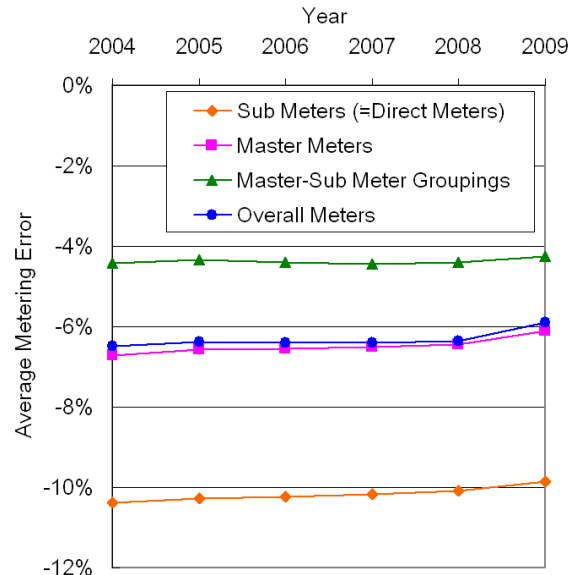


Fig.11. Metering Errors of TWD from 2004 to 2009

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